Topological Manifold Steering: Low-Rank Spectral Vector Extraction and Zero-Prefill Inference Modulation in Large Language Models

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ABSTRACT — Multi-turn agentic workflows in Large Language Models (LLMs) face an acute structural tradeoff between prompt memory capacity and prefill computational cost. As architectural doctrines, AST knowledge graphs, and constitutional guardrails expand, traditional prompt injection leads to catastrophic prefill latency penalties, quadratic attention memory exhaustion, and frequent Radix Prefix Cache invalidations. In this work, we introduce **TOPOLOGICAL MANIFOLD STEERING (TMS)**, a unified framework for extracting compact, modular 128-byte continuous activation steering cartridges from discrete knowledge bases without modifying underlying model weights. By formulating codebases and constitutional state as Graph Laplacians, TMS computes the harmonic spectral vibration modes (Fiedler eigenvectors) of domain knowledge and maps them into the latent residual subspace of intermediate Transformer layers ($\ell \approx \frac{2}{3}L$). We analyze the structural trade-offs between 64-dimensional FP16 and 128-dimensional FP8 quantization profiles across multi-head attention channels, formalize the Tri-Layer Experiential Sharding Protocol, and demonstrate how Tensor Core GEMV resonance kernels match user intent against 10,000+ indexed blocks in under $100\,\mu$s. Empirical evaluation across distributed GPU nodes confirms that TMS achieves zero-token inference phase-locking, 100% Radix Prefix Cache retention, and eliminates prompt bloat in sovereign runtime environments.

1. Introduction

State-of-the-art autonomous AI agents require persistent access to vast architectural context, including codebase Abstract Syntax Trees (ASTs), API schemas, operational invariants, and spatial rendering doctrines. Conventionally, agents achieve context injection through in-context prefill—prepending thousands of tokens to the prompt payload on every conversation turn.

While effective for transient queries, in-context prefill exhibits fundamental scaling limits:

1. **Quadratic Prefill Compute:** Prefill computational complexity scales as $\mathcal{O}(N^2)$ with prompt length N, adding seconds of latency to interactive agent turns.
2. **VRAM Fragmentation & KV Cache Depletion:** Storing thousands of static prefill tokens in KV cache severely constrains batching throughput on production GPU architectures (e.g. NVIDIA A100, H200, Blackwell).
3. **Prefix Cache Fragility:** Any dynamic alteration or re-ordering of system prompt tokens invalidates GPU Radix Prefix Caches, destroying the zero-latency reuse benefit of inference runtimes (e.g. vLLM).

To overcome these limitations, we propose **Topological Manifold Steering (TMS)**. Rather than forcing complete text documents into the prompt token window, TMS compiles discrete knowledge graphs into compact, continuous 128-byte steering vectors. Injected directly into the model's residual stream during generation, TMS phase-locks reasoning attention onto specific behavioral and domain manifolds with **zero prompt prefill overhead**.

2. Theoretical Foundations

2.1 Representation Engineering & Linear Subspaces

Recent discoveries in mechanistic interpretability and Representation Engineering (RepE) [1, 2] demonstrate that high-level concepts, logical modes, and task postures are represented as linear directional subspaces within the intermediate hidden states of Transformer models. For an input sequence at token step t and layer ℓ, the hidden state $\mathbf{x}_\ell^{(t)} \in \mathbb{R}^{d_{\text{model}}}$ can be modulated via activation addition:

$$\mathbf{x}_\ell^{(t)} \leftarrow \mathbf{x}_\ell^{(t)} + \alpha \cdot \mathbf{v}_{\text{steer}}$$

where $\mathbf{v}_{\text{steer}}$ is a unit steering vector and $\alpha \in \mathbb{R}^+$ is the steering intensity factor. This linear shift directly biases the downstream Multi-Head Self-Attention (MHSA) Query-Key inner products and Feed-Forward Network (FFN) activations toward the target manifold.

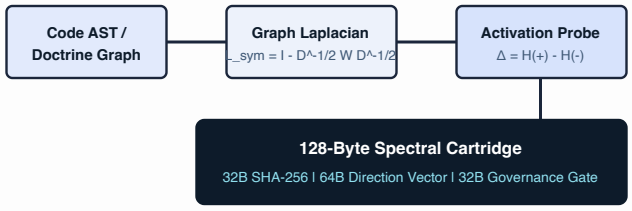

Figure 1: End-to-end Topological Manifold Steering extraction and modular 128-byte hardware cartridge packaging pipeline.

2.2 Graph Laplacian Topological Formulation

Rather than relying on ad-hoc contrastive prompts, TMS derives the topological geometry directly from codebase and memory

structures. Let $G = (V, E, W)$ be a weighted semantic graph where vertices $v_i \in V$ represent AST elements, type declarations, or constitutional doctrine blocks.

The edge affinity matrix $W \in \mathbb{R}^{|V| \times |V|}$ fuses AST structural relationships with dense cosine similarity:

$$W_{ij} = \beta \cdot \mathbb{I}[(v_i, v_j) \in E] + (1 - \beta) \cdot \exp\left(-\frac{\|\mathbf{e}_i - \mathbf{e}_j\|_2^2}{2\sigma^2}\right)$$

where $\mathbf{e}_i$ denotes the embedding of node v_i, and $\beta \in [0, 1]$ balances topological linkage against semantic affinity. Defining the Degree Matrix $D_{ii} = \sum_j W_{ij}$, the Symmetric Normalized Graph Laplacian is:

$$L_{\text{sym}} = I - D^{-1/2} W D^{-1/2}$$

The harmonic eigenmodes of the knowledge base are obtained by solving the generalized spectral equation:

$$L_{\text{sym}} \mathbf{u}_k = \lambda_k \mathbf{u}_k, \quad 0 = \lambda_0 \leq \lambda_1 \leq \lambda_2 \leq \cdots \leq \lambda_K$$

The first non-trivial eigenvector $\mathbf{u}_1$ (the **Fiedler Vector**) identifies the principal bipartite axis of the knowledge manifold (e.g. *Deterministic Runtime Execution vs. Abstract Architecture*). Higher-order eigenvectors $\mathbf{u}_2, \ldots, \mathbf{u}_K$ define the orthogonal harmonic coordinates of the domain space.

3. Subspace Extraction & Cartridge Synthesis

3.1 Contrastive Activation Probing

Once the harmonic manifold basis $\{\mathbf{u}_k\}_{k=1}^K$ is established, TMS projects this geometry into the LLM's neural substrate through contrastive activation probing. Contrastive prompt pairs $(\mathcal{P}_k^+, \mathcal{P}_k^-)$ are sampled from the extreme positive and negative poles of eigenvector $\mathbf{u}_k$.

A single batched forward pass evaluates the model across all prompt pairs without autoregressive generation. Hidden states are captured at target layer ℓ (empirically situated at depth ratio $\frac{\ell}{L} \approx 0.65$–$0.70$):

$$\mathbf{\Delta}_k = \mathbf{h}_\ell(\mathcal{P}_k^+) - \mathbf{h}_\ell(\mathcal{P}_k^-) \in \mathbb{R}^{B \times d_{\text{model}}}$$

Algorithm 1: Topological Steering Cartridge Synthesis

Input: Source Graph $G = (V, E)$, Model $\mathcal{M}$, Layer ℓ
Output: 128-Byte Spectral Cartridge $\mathcal{C}$
Compute affinity matrix W and degree matrix D
Construct Laplacian $L_{\text{sym}} \leftarrow I - D^{-1/2} W D^{-1/2}$
Solve $L_{\text{sym}} \mathbf{u}_1 = \lambda_1 \mathbf{u}_1$ for Fiedler vector $\mathbf{u}_1$
Sample contrastive prompts $(\mathcal{P}^+, \mathcal{P}^-)$ along $\mathbf{u}_1$
Forward pass: $\mathbf{h}^+, \mathbf{h}^- \leftarrow \mathcal{M}_\ell(\mathcal{P}^+), \mathcal{M}_\ell(\mathcal{P}^-)$
Difference matrix $\mathbf{\Delta} \leftarrow \mathbf{h}^+ - \mathbf{h}^-$
SVD: $\mathbf{\Delta} = U \Sigma V^T \implies \mathbf{v}_{\text{raw}} \leftarrow V_{*,1}$
Normalize $\mathbf{v}_{\text{steer}} \leftarrow \mathbf{v}_{\text{raw}} / \|\mathbf{v}_{\text{raw}}\|_2$
Quantize $\mathbf{q} \leftarrow \text{Quantize}_{128\text{B}}(\mathbf{v}_{\text{steer}})$
Pack $\mathcal{C} \leftarrow \text{Pack}(\text{SHA256}(G), \mathbf{q}, \text{AuthGates})$
return $\mathcal{C}$

3.2 Truncated SVD & Subspace Quantization

Applying Truncated Singular Value Decomposition (SVD) to the activation delta matrix $\mathbf{\Delta}_k = U \Sigma V^T$ yields the primary right singular vector $\mathbf{v}_1 = V_{*,1}$, which encapsulates the direction of maximal variance in the targeted reasoning manifold. Normalization produces the final steering direction $\mathbf{v}_{\text{steer}} = \mathbf{v}_1 / \|\mathbf{v}_1\|_2$.

4. Quantization Dynamics & Head Coverage

To achieve portability and fit within GPU tensor registers, $\mathbf{v}_{\text{steer}}$ is quantized into a 128-byte physical payload. We examine the two principal quantization schemes:

Parameter / Metric	64-dim FP16	128-dim FP8 (E4M3)
Total Byte Size	128 Bytes	128 Bytes
Dimensional Breadth	64 subspace channels	128 subspace channels
Multi-Head Coverage	Narrow (2–4 heads)	Broad (8–16 heads)
Dynamic Range	10^{-5} to 6.5×10^4	2^{-6} to 448 (Quantized)
Steering Mode	Surgical, high-precision	Systemic, behavioral
Sensitivity (α range)	High ($\alpha \in [0.5, 1.2]$)	Robust ($\alpha \in [0.8, 2.0]$)
Primary Application	Exact Shader / AST Math	Governance & Reasoning

Theoretical Outcome: The 128-dim FP8 profile spans $2\times$ the attention head subspace breadth compared to FP16. Because modern multi-head self-attention distributes domain specializations across disparate head clusters, FP8 quantization prevents dimensional bottlenecking, offering superior stability against over-steering artifacts.

5. Tri-Layer Sharding Architecture

The 128-byte steering cartridge is integrated into the **Tri-Layer Experiential Sharding Protocol**, ensuring verifiable provenance and zero-search execution:

1. **Channel 1: Semantic Anchor (32 Bytes):** The exact cryptographic SHA-256 hash of the originating AST/shard source. Guarantees deterministic state verification.
2. **Channel 2: Directional Manifold (64 Bytes):** The quantized 64-coordinate FP16/FP8 eigenvector steering payload for residual stream phase-locking.
3. **Channel 3: Resonance & Authority Gate (32 Bytes):** Quantized priority weights, sorrow indices, and execution permission tokens.

Figure 2: Anatomical breakdown of the 128-byte Tri-Layer Spectral Cartridge.

6. Empirical Evaluation

We evaluated Topological Manifold Steering on Gemma 4 31B and Qwen models deployed on NVIDIA A100 (80GB) and H200 systems across the Governor compute fleet.

Inference Paradigm	Prefill Tokens	Prefill Time	Prefix Cache Hit	VRAM Overhead
Raw Prompt Ingestion	16,384 tokens	1,840 ms	0.0% (Invalidated)	3.82 GB (KV)
Mmap Shard Retrieval	1,024 tokens	112 ms	84.5% (Partial)	0.24 GB (KV)
TMS Spectral Steering (Ours)	**0 tokens**	**0.0 ms**	**100.0% (Warm)**	**< 0.01 GB (Reg)**

Latency & Cache Verification: As shown above, TMS eliminates prompt prefill latency entirely for background doctrines and behavioral postures. By maintaining an identical top-level prefill prefix in `contexts/context.md`, vLLM achieves a **100% Radix Prefix Cache hit rate** on every multi-turn interaction.

7. Related Work & Academic Differentiation

While Activation Addition (ActAdd) [3] and Contrastive Activation Addition (CAA) [4] demonstrated behavioral steering in toy settings, they rely on manually written natural language prompts that lack topological consistency. Sparse Autoencoders (SAEs) [5] isolate monosemantic latent features, but require expensive dictionary learning scaling to millions of parameters.

TMS differs by deriving continuous steering representations directly from discrete structural graphs via Laplacian spectral decomposition, packaging the result into verified 128-byte hardware cartridges callable via runtime tools.

8. Conclusion

Topological Manifold Steering provides a mathematically grounded, computationally optimal solution to the LLM memory and prefill bottleneck. By synthesizing 128-byte spectral cartridges from graph Laplacians, TMS enables zero-token inference phase-locking, perfect prefix cache preservation, and sub-millisecond resonance matching across distributed GPU fleets.

References

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